Aim of the applet:
The aim of this applet is to plot planes and lineations in a lower Hemisphere Schmidt Net using the Lambert (equal area) projection, and a Wulffs Net (Equal angle). It can also be used to calculate the angles between two planes, between a plane and a lineation, between two lineations and between a lineation and a plane intersection line.

Input:
The applet can plot two planes and two lineations. The input consist therefore of the dip direction and dip of planes 1 and 2, and the azimuth and the plunge of lineation 1 and 2 respectively.

Output:
The output consists of the projection of the lineation 1 and 2 and both the pole and the great circle of planes 1 and 2 on a lower hemisphere Schmidt Net as well as in a lower hemisphere Wulffs Net. The output further offers the Azimuth (°) and Plunge (°) of the Planes 1 and 2 intersection line, as well as the angles between lineations, the angles between the planes and the lineations, the angle between the planes and the angles between the lineations and the plane intersection line. All these angles are in degrees and are the smallest angles. This means that angles between lineations never exceed 180° and angles between planes and lineations and planes never exceed 90°. For example, the angle between a plane with orientation 000/10 and a lineation 180/10 is 20° and not 160° (see figure 1).

Coordinates systems:
Different coordinate systems are used for input, calculation and projection of objects and angles in this applet. The input data format is the Polar coordinate system, with Azimuth and Plunge as the components. The Azimuth is the angle in degrees from the North line, which is defined as straight up in the diagrams in this applet. The Plunge is the deviation in degrees from the horizontal plane. Note when planes are concerned, the Azimuth describes the Dip Direction and the Plunge equals the Dip of the plane.

For the calculations, the right-handed 3D Cartesian vector coordinate system is used, where the following rules apply: the vectors have three elements, (x,y,z), the Cartesian axes are orthogonal and the x direction is up (north), y is right (east) and z is down.

For the projection of the objects, a different, 2D Cartesian coordinate system is used, where x is right (east) and y is up (north).

Note to Excel
In Excel, all angles are calculated as radian, while the output of the applet and in this script the angle will be described in degrees. Note that radians can be turned into degrees by dividing the radian value by (π/180).
**Turning lineation and planes orientations into vectors:**

Being a line, a lineation can be transformed into a vector relatively easy. The vector is calculated using the formulas:

\[
x = \cos \varphi \cdot \cos \alpha \\
y = \cos \varphi \cdot \sin \alpha \\
z = \sin \varphi
\]

Where \( \alpha \) is the azimuth in degrees, and \( \varphi \) is the plunge, in °. A major assumption in this step is that the azimuth and plunge represent a line of the length 1. That means that the \( \cos \) (Plunge) represents the length of the line in the xy-plane (V').

In this applet, planes are initially characterized by their pole (a line which is perpendicular to the plane). Making a pole out of a plane is done using the formulas:

Pole azimuth = Plane azimuth +180
Pole Dip = 90 – Plane dip

This line can then be turned into a vector using formulas (1.1).

Note that since all vectors are recalculated into unit vectors (vectors with magnitude 1) before they are used further in this applet.

**Calculating angles between lines and planes:**

The Scalar Product of two vectors is a number (scalar) and is defined as \( A \cdot B = |A||B| \cos \theta \) and as \( A \cdot B = A_x B_x + A_y B_y + A_z B_z \), where \( |A| \) is the length/magnitude of vector A (=\( \sqrt{A_x^2 + A_y^2 + A_z^2} \)), \( |B| \) is the length of vector B, and \( \theta \) is the angle between vectors A and B. Combining these formulas produces:

\[
\theta = \cos^{-1} \left( \frac{A \cdot B}{|A||B|} \right)
\]

\[
\theta = \cos^{-1} \left( \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 A_x^2} \sqrt{B_x^2 B_x^2}} \right)
\]

Using this formula, the calculation of the angle between all lines is possible.

For the calculation of the angle between two planes, the same formula can be used, but the poles to the plane needs to bee used as input vector.

For the calculation of the angle between a line and a plane, the pole vector needs to bee used as input for the plane. Since the pole vector by definition has an angle of 90° to the plane, the angle between a line and a plane (\( \theta' \)) is defined as \( \theta' = 90 - \theta \).

**Calculating the intersection between two planes:**

The intersection line between two planes is an important object as it can represent the fold axis if the planes represent the two sides of the fold, or the delat lination, that is the intersection line between bedding and cleavage. To calculate the plane intersection line, the Vector Product is used. The vector product is defined as:

\[
A \times B = (iA_x + jA_y + kA_z) \times (iB_x + jB_y + kB_z)
\]

\[
A \times B = i(A_y B_z - A_z B_y) + j(A_z B_x - A_x B_z) + k(A_x B_y - A_y B_x)
\]
Where \(i, j, k\) are unit vectors in the direction \(x, y\) and \(z\) respectively. These define the vector \(v\), which is a vector perpendicular to both vector \(A\) and \(B\), and therefore parallel to the intersection of these two planes.

To calculate the azimuth of the plane intersection line, the formulas are:

\[
\alpha = \tan^{-1}\left(\frac{\wedge \hat{v}_y}{\wedge \hat{v}_x}\right)
\]

\[
\varphi = \tan^{-1}\left(\frac{\wedge \hat{v}_z}{\sqrt{\hat{v}_x^2 + \hat{v}_y^2}}\right)
\]

(1.4)

Here \(\hat{v}_x\), \(\hat{v}_y\) and \(\hat{v}_z\) are the components of the vector \(v\).

Calculating the great circle of a plane:

The determination of the great circle of a plane consists of two steps (see figure 3). First the strike component of the plane needs to be determined, and then this vector needs to be rotated around the pole of the plane, describing a semicircle. The strike component is a horizontal vector parallel to the strike of the plane (that is Dip Direction± 90°). Since this vector is horizontal, the \(z\) value of this vector is 0. To determine the Strike component, the vector product of the pole of the plane and the vertical vector (= 0*i, 0*j, 1*k) was determined. Since the vector product always returns a vector perpendicular to the input vectors, the resultant vector will be parallel to the strike. When this vector is normalized (when a unit vector is made of it) it represents the vector projection of the strike on a lower hemisphere projection with radius 1.

To create the Great circle, this point is now rotated 180° around the pole in 3 degree increments. These 60 points are then combined to form the great circle.

The strike vector is rotated using a rotation matrix \(M\). In 3D-space, \(M\) is a 3*3 matrix of the form:
Here $\theta$ is rotation angle, and $v_x$, $v_y$ and $v_z$ are the components of the vector $v$ about which the rotation needs to be performed (in this case, the pole vector of the plane). A new rotation matrix is required for each of the 60 points that define the great circle.

The formula to rotate a vector (for example the strike vector $S$) into its rotated equivalent ($S'$) is:

$$ S' = MS $$

Since this operation is simple rotation, the rotated vectors still have the magnitude 1, and therefore, in 3D define a semicircle.

**Projection of the vectors:**

Because all the vectors in the great circle, the pole and lineation vectors are unit vectors, all together they describe a sphere with the diameter 1. Projection of these vectors on a horizontal plane (by simple plotting the x and y values of the vector in a xy-plane) does however have no value, as that represents neither the equal angle (Wulffs Net), nor the equal area (Schmidt Net) projection (note the differences between the left and right panels of figure 6). The principles of the equal area or Lambert projection are given in figure 4.

![Figure 4: The principles of the Lambert Projection, see text for details.](image)

Here point B represents the endpoint of a vector with unit length 1. The plane underneath the lower hemisphere represents the plane of the Lambert Projection, and point X in this plane is the projection of point B on it. This is done in such a way that $AB = AX$. This means this not a true projection, rather a rotation of point B into point X. The distance $AX$ is defined as:

$$ AX = \sqrt{2R \sin \left( \frac{\pi - \phi}{4} \right)} $$
Here $\varphi$ is the plunge and $R$ is the radius of the projected sphere, in this case, $R = 1$.

To project lineation and poles on the xy-plane, the azimuth and plunge of these objects are used in the formulas (note that the direction of x and y is different than in the vector coordinate system):

$$x = \sin(\alpha) \sqrt{2 \sin\left(\frac{\pi - \varphi}{4}\right)}$$

$$y = \cos(\alpha) \sqrt{2 \sin\left(\frac{\pi - \varphi}{4}\right)}$$

Where $\alpha$ is the azimuth and $\varphi$ is the plunge. These formulas are also used to plot the great circles and the intersection vector.

All data was also plotted in a Wulffs Net. The principles of the stereographic projection, on which the Wulffs Net is based, are given in figure 5.

Here $A'$ is the projection of point $A$, and the distance $0A'$ is defined as:

$$(1.9) \quad 0A' = R \left( \tan \frac{90 - \varphi}{2} \right)$$

Here $R$ is the radius of the projection circle (1 in our case), and $\varphi$ is the plunge. To project lineation and poles on the xy-plane, the azimuth and plunge of these objects are used in the formulas (note that the direction of x and y is different than in the vector coordinate system):

Figure 5: The principles of the Stereographic Projection.
\[
\begin{align*}
    x &= \sin(\alpha) \cdot \left(\tan \frac{90 - \varphi}{2}\right) \\
    y &= \cos(\alpha) \cdot \left(\tan \frac{90 - \varphi}{2}\right)
\end{align*}
\]  

(1.10)

Where \(\alpha\) is the azimuth and \(\varphi\) is the plunge. These formulas are also used to plot the great circles and the intersection vector.

**Note to projecting great circles in the applet:**

If the vector \(x\) coordinate is smaller than 0, the calculation of the \(xy\)-projection-coordinates produces wrong values, in such a way that the great circle does not enter into the negative \(y\) area of the Schmidt net but projects an additional, conjugate great circle fragment (see figure 6, right panel). This is most likely the result of the back calculation to azimuth and plunge, which is required before the projection coordinates can be plotted.

The problem is solved by simply taking the negative \(x\) and \(y\) projection coordinates, if the vector \(x\) coordinate is smaller than 0.

**Figure 6:** Panels from the excel sheet showing the difference between vector projection and Schmidt Net projection, as well as the result of a smaller than 0 value for the vector \(x\) coordinate on the projection of the great circle.